Not-So-Cleansing Recessions

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Motivation

- *Liquidationist view of business cycles:*

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  - typically *good reallocation*:
    - \+ unproductive firms/obsolete capital exit
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⇒ *Cleansing effects of recessions*

Speed up the replacement process/ improve the average efficiency

Caballero & Hammour (1994)

Motivation

Long tradition in economics

Stiglitz (1993):

There is a famous aphorism that in every cloud, there is a silver lining. The alleged silver lining in the cloud of an economic recession is the "shake-out" effect. As firms face declining profits and cash reserves, they typically act to cut out fat, to fire unnecessary workers, and to restructure the firm to make it "leaner and meaner".

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Schumpeter (1934):
“errors and misbehaviour should be abnormally frequent in prosperity [...] everything that is unsound for either reason shows up when prices break and credit ceases to expand in response to decreased demand for it.”

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Long tradition in economics

Schumpeter (1934):

recessions [...] "are but temporary. They are the means to reconstruct each time the economic system on a more efficient plan. But they inflict losses while they last, drive firms into the bankruptcy court, throw people out of employment, before the ground is clear and the way paved for new achievement of the kind which has created modern civilization and made the greatness of this country."

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Recessions are costly in the short run but good in the long run

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Old point ...

- Scarring effects of recessions (workers are worse off) – Ouyang (2009)
- Reduced innovation investment (slower long-run growth) – Stiglitz (1993)
- Sullying effects (productive firms killed in infancy) – Barlevy (2002)

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... with a twist: depends on how much agents value varieties

Model

Setup (1/2)

- Industry in monopolistic competition.
- Firm (variety) technology: $y = zl$.
  - Firms must pay a fixed cost f^c, in units of labor, to produce.
  - small industry: can hire labour (numeraire) at $w = 1$.
- Entry: ex-ante homogeneous firms must pay a fixed cost of entry f^e, also in units of labor, to enter and draw productivity $z \sim \mu^0$.

Setup (2/2)

- Competitive intermediary combines varieties into final good Y

$$Y = \left[\int y(z)^{\frac{\sigma-1}{\sigma}}\mu(z)dz\right]^{\frac{\sigma}{\sigma-1}}$$

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with $\int \mu(z)dz = M$. (Dixit and Stiglitz, 1975; Ethier, 1982; Benassy, 1996)

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Variety Effect

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Y^{CES}

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- Representative household with exogenous total income $\mathcal{I}$ and $\mathcal{U}(Y)$, $\mathcal{U}' > 0$.

Firm Behaviour

Firm problem

$$\max_{p(z)} \pi(z,\mu) = y(p(y) - 1/z) - f^c \quad \text{s.t.}$$

$$y(z,\mu) = p(z)^{-\sigma} P(\mu)^{\sigma-1} \mathcal{I} M^{q(\sigma-1)-1},$$

Firm problem

Total Income

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Entry problem

$$f^e \geq \mathbb{E}_0[\max\{\pi(z,\mu), 0\}]$$

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Expectation wrt μ_0

Current Distribution

$$f^e \geq \mathbb{E}_0[\max\{\pi(z,\mu), 0\}]$$

Firm Behaviour

At the optimal price $p^{\star}(z)$, profits are:

$$\pi(z, \mu) = \frac{\mathcal{I}}{\sigma} \frac{z^{\sigma-1}}{\int z^{\sigma-1} \mu(z)} - f^c$$

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Aggregate Profits

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fraction going to firm z

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Since π is strictly increasing in z, $\exists! \underline{z}$:

$$0 = \pi(\underline{z}, \mu)$$

$\Rightarrow$ firms with $z < \underline{z}$ exit.

Equilibrium

Equilibrium is given by a triplet $\underline{z}$, E, $\mu(z)$ such that

$$0 = \pi(\underline{z}, \mu)$$
$$f^e \geq \mathbb{E}_0[\max\{\pi(z, \mu), 0\}]$$

and after entry

$$\mu(z) = (\mu^I(z) + E\mu^0(z))\mathbb{I}_{\{z \geq \underline{z}\}}$$

where μ^I is the distribution of incumbents with $\int \mu^I(z)dz = I$.

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Conditional on survival

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Remarks

- no exogenous death shock nor idiosyncratic productivity fluctuation
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  $\Rightarrow$ absent shocks, only exit upon entry and draw $z < \underline{z}$.
- no financial markets $\Rightarrow$ firms exit immediately if $\pi < 0$
- the instantaneous entry game is equivalent to an iterative one.

Equilibrium characterization (1/2)

Entry

$$\frac{f^e}{f^c} = \int_{\underline{z}}^{\infty} \left[\left(\frac{z}{\underline{z}} \right)^{\sigma-1} - 1 \right] \mu^0(z) dz$$

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$\underline{z}$ does NOT! depend on Incumbents

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cutoff vs expected entrant

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cutoff vs expected entrant

Incumbent vs expected entrant

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If there are not incumbents: $I = 0$

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$= \dfrac{M}{Pr(z > \underline{z})}$

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$= \mu^0(z)$

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$\mu^0(z)$

= 1

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Note:

⇒ incumbents reduce the space for entry

⇒ when $E > 0$, $\underline{z}$ is independent of incumbents

Equilibrium characterization (2/2)

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Pins down $\underline{z}$

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$\underline{z}$ DOES depend on incumbents

Business Cycles

Business Cycle definition

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Split time into 3 phases:

- $\tau = 1$: fixed costs equal f_l^c
- $\tau = 2$: fixed costs unexpectedly increase to $f_h^c > f_l^c$
- $\tau = 3$: fixed costs revert to f_l^c

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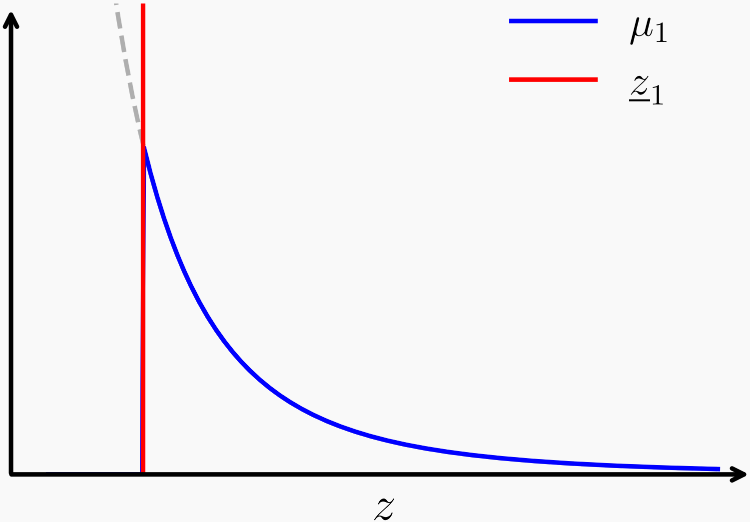
Goal: compare phase 1 to phase 3.

Business Cycle in Pictures



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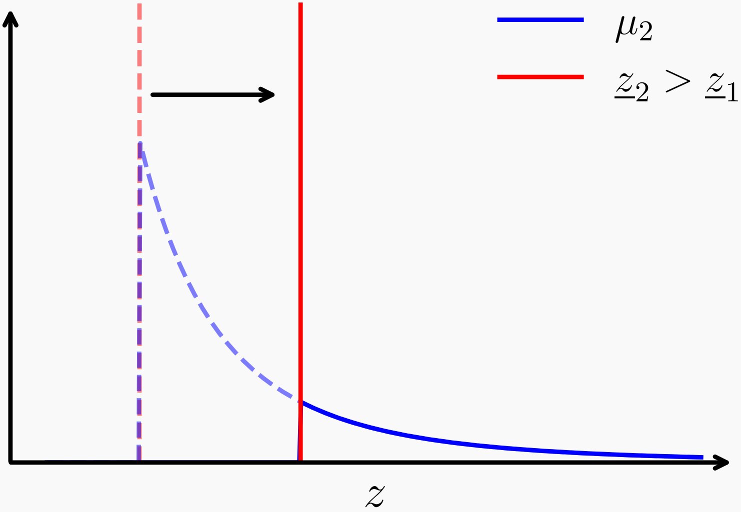

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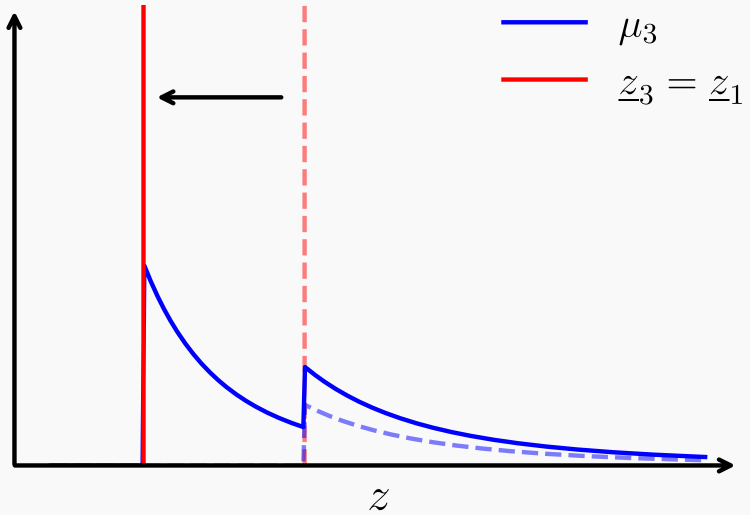

Business Cycle in Maths

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↳ $:= Z_1$

Aggregate TFP

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Demand for production labour

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Variety Effect

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$:= Z_1$

Aggregate TFP

Business Cycle in Maths

Suppose we start from a steady state with $\underline{z}_1$, μ_1 integrating to M_1 $(E_1 = 0)$

$$Y_1 = M_1^{q-\frac{1}{\sigma-1}} L_1^d \left(\int z^{\sigma-1} \mu_1(z) dz \right)^{\frac{1}{\sigma-1}}$$

Suppose that $f^c \uparrow$

Business Cycle in Maths

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$\Rightarrow$ *Exit*

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Business Cycle in Maths

Suppose that $f^c \downarrow$, back to original level

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$\Rightarrow$ *Entry*

But recall

$$\frac{f^e}{f^c} = \int_{\underline{z}}^{\infty} \left[\left(\frac{z}{\underline{z}} \right)^{\sigma-1} - 1 \right] \mu^0(z) dz$$

Business Cycle in Maths

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What happens to the number of firms M?

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This equation has to hold

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Welfare Effects of Cycles

$$\Delta \log Y = q\Delta \log M + \Delta \log L^d + \frac{1}{\sigma - 1}\Delta \log \bar{z}$$

Welfare Effects of Cycles

Variety effect ↓

Selection effect ↑

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Welfare Effects of Cycles

Variety effect

Aggr. TFP = 0

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+/−

Welfare Effects of Cycles

Proposition

The effect of recessions in PE is given by

$$\Delta \log Y = \left(q - \frac{1}{\sigma - 1}\right) \Delta \log M + \Delta \log L^d + \Delta \log \mathcal{Z}$$

where

$$\Delta \log M < 0 \;\; \text{and} \;\; \Delta \log \mathcal{Z} = 0.$$

Hence,

$$\Delta \log Y \gtreqless 0 \Leftrightarrow q \lesseqgtr q^{CES} \equiv \frac{1}{\sigma - 1}.$$

Intuition

Start from equilibrium aggregate output:

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- **CES,** $q = \frac{1}{\sigma - 1}$ ⇒ **variety and selection perfectly offset each other.**

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- **CES,** $q = \frac{1}{\sigma - 1} \Rightarrow$ **variety and selection perfectly offset each other.**
- Trade-off firm selection and loss of varieties is welfare-relevant *only away from CES*.

General Equilibrium

General Equilibrium

Same economy but

1. Endogenous income: $\mathcal{I} = R = (w)\bar{L} + \Pi$
2. Labor market clearing (industry is not small):

$$L^d + Mf^c + Ef^e = \bar{L}.$$

Welfare Effects of Cycles in GE

Proposition

The effect of recessions in GE is given by

$$\Delta \log Y = \left(q - \frac{1}{\sigma - 1}\right) \Delta \log M + \Delta \log L^d + \Delta \log \mathcal{Z}$$

where

$$\Delta \log M < 0, \; \Delta \log L^d > 0 \; \textit{and} \; \Delta \log \mathcal{Z} > 0.$$

There exists a unique $q^\star > q^{CES}$ *for which* $\Delta \log Y = 0$. *Furthermore*

$$\Delta \log Y < 0 \Leftrightarrow q > q^\star.$$

Intuition (1/2)

$$\Delta \log Y = \left(q - \frac{1}{\sigma - 1} \right) \Delta \log M + \Delta \log L^d + \Delta \log \mathcal{Z}$$

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In GE saving labour $\Rightarrow$ some extra entry

1. After $f^c \uparrow$ and $\downarrow$, economy saves on fixed production costs as $M \downarrow$

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In GE saving labour $\Rightarrow$ some extra entry

1. After $f^c \uparrow$ and $\downarrow$, economy saves on fixed production costs as $M \downarrow$
2. As if a small endowment effect $\Rightarrow M \downarrow$ less than in PE

Intuition (2/2)

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Intuition (2/2)

$$\Delta \log Y = \left(q - \frac{1}{\sigma - 1}\right) \Delta \log M + \Delta \log L^d + \Delta \log \mathcal{Z}$$

With CES $q = \frac{1}{\sigma - 1} \Rightarrow \Delta \log Y > 0$

Business Cycle Depth

Consider now recessions of different intensities f_h^c:

$\tau = 1$: fixed costs equal f_l^c

$\tau = 2$: fixed costs unexpectedly increase to f_h^c

$\tau = 3$: fixed costs revert to f_l^c

Business Cycle Depth

Recall: **We are after long-run effects**

$\Rightarrow$ deeper crises might be better in the long run.

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Recessions trade off *variety losses* with *cleansing effects*.

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A deeper recession necessarily generates more exit along the transition but not obvious on long-run M

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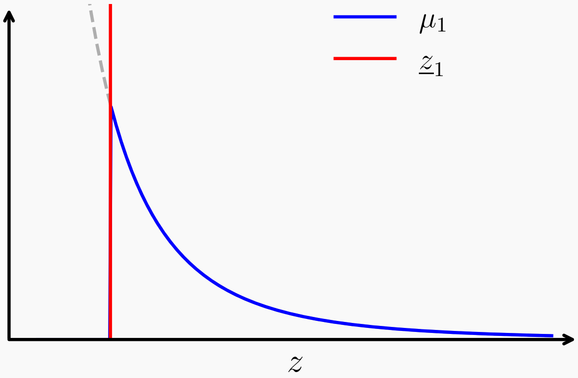
$\Rightarrow$ deeper crises might be better in the long run.

Recessions trade off *variety losses* with *cleansing effects*.

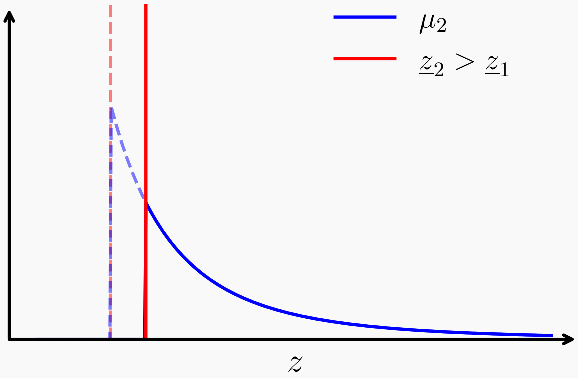
A deeper recession necessarily generates more exit along the transition but not obvious on long-run M

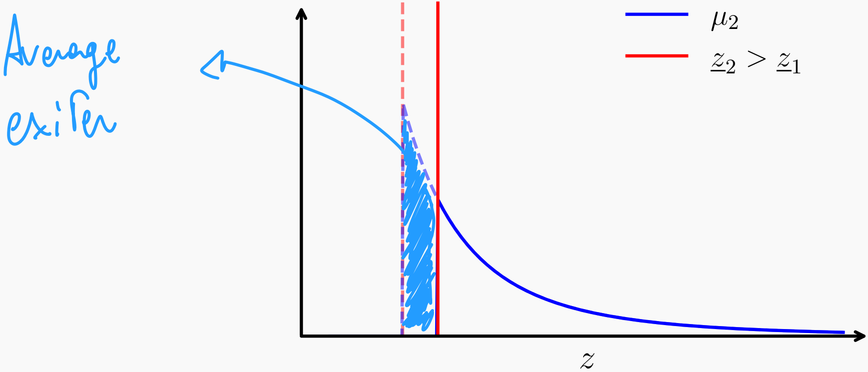
Consider two crises: one small, one large.

A small recession

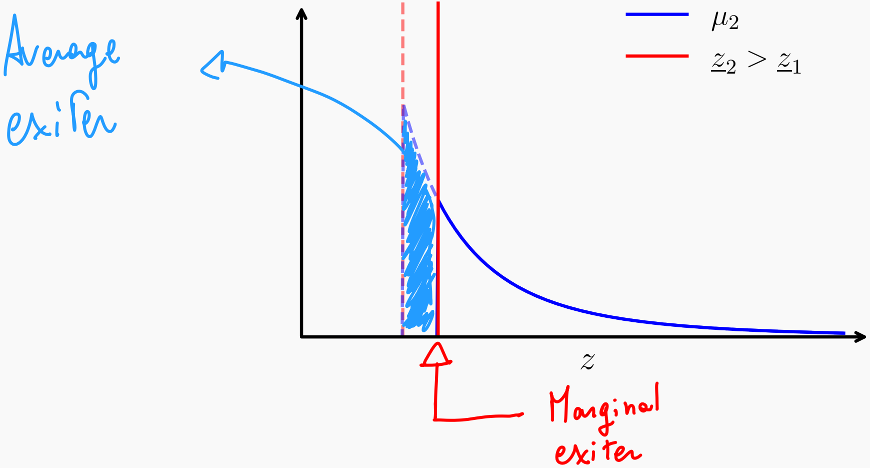


A small recession

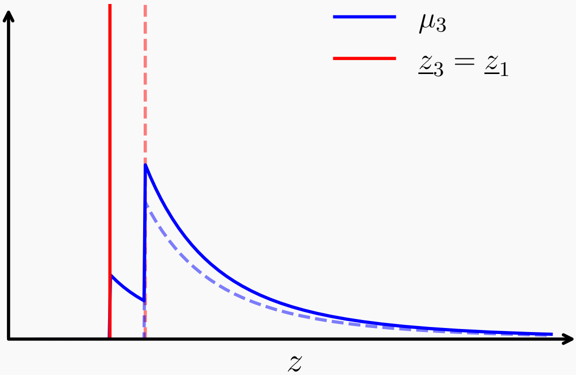



Average exiter
μ_2
$\underline{z}_2 > \underline{z}_1$
z

A small recession



A small recession



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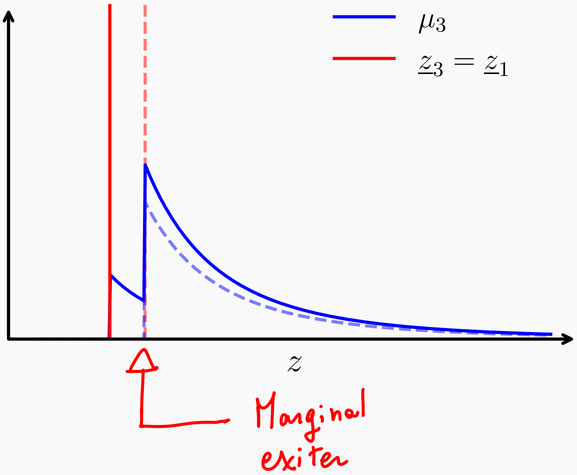

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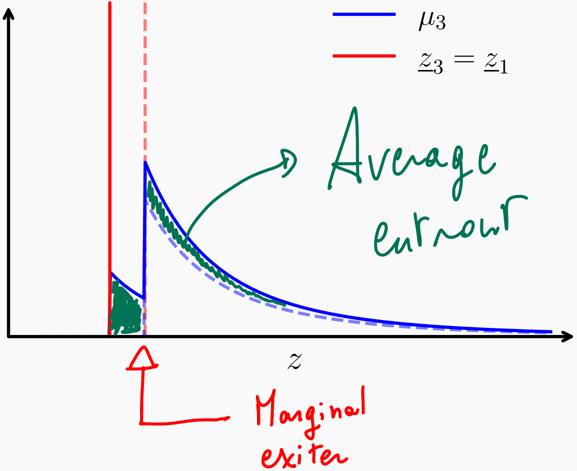

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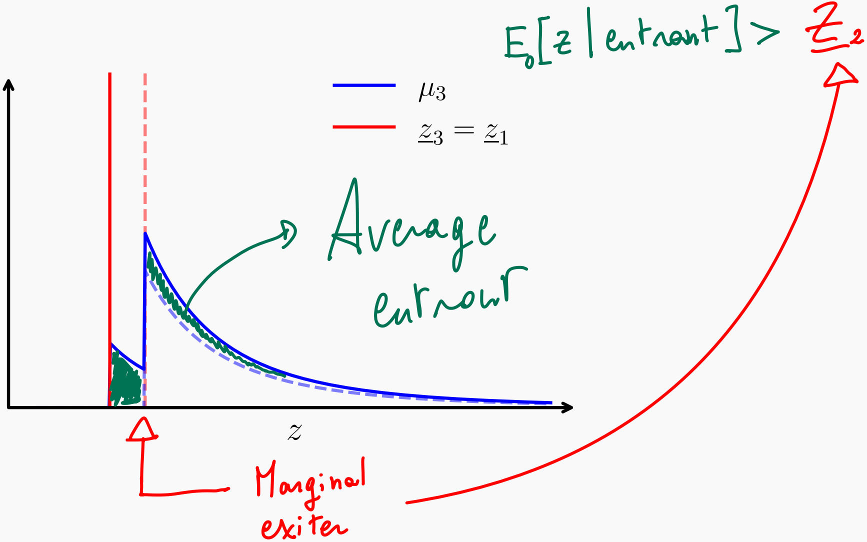

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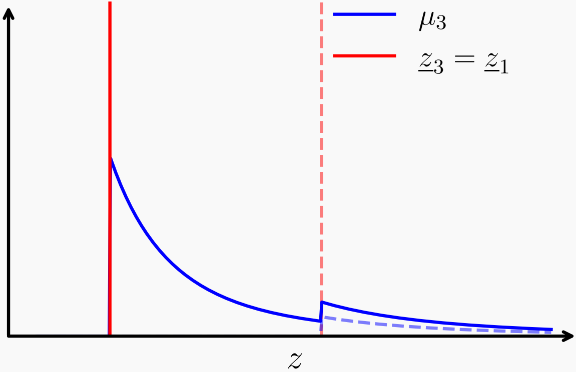

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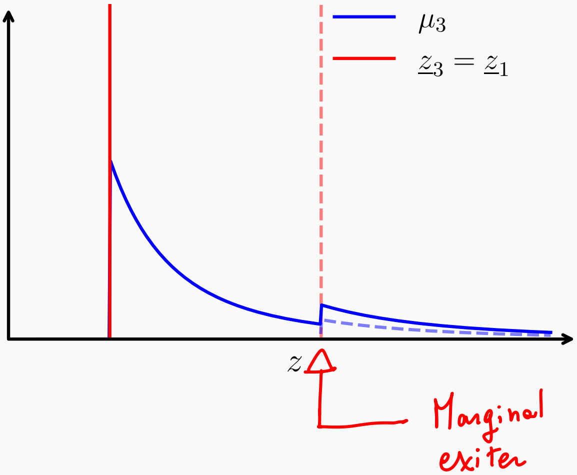
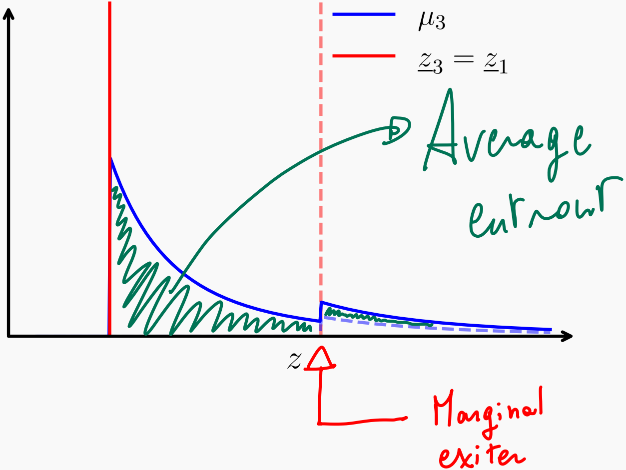
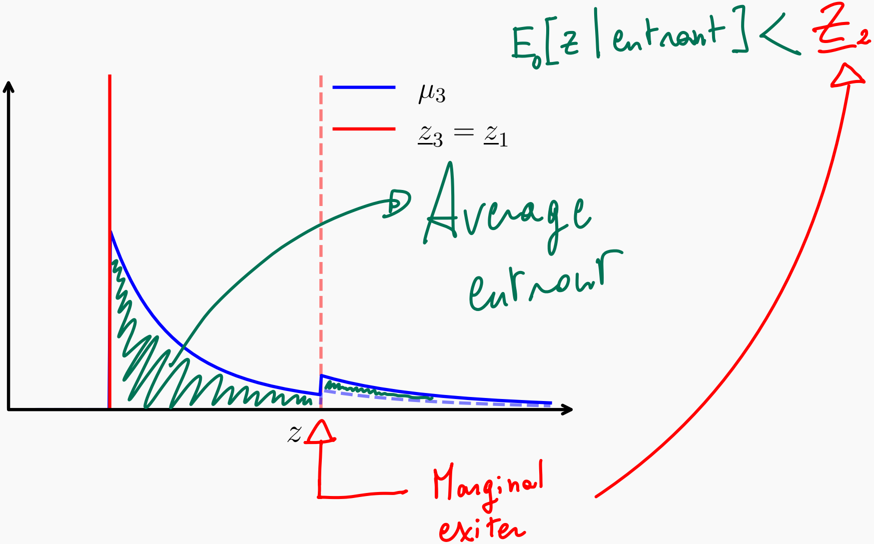


A large recession





$E_0[z \mid \text{entrant}] < \underline{z}_2$
μ_3
$\underline{z}_3 = \underline{z}_1$
Average entrant
z
Marginal exiter

Business Cycle Depth

Start again from equilibrium post-recession aggregate output:

$$Y_3 = M_3^{q-\frac{1}{\sigma-1}} \underbrace{L_3^d \left(\int z^{\sigma-1} \mu_3(z) dz \right)^{\frac{1}{\sigma-1}}}_{Y_3^{CES}}$$

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Decomposition of the Elasticity - M_3 to f_h^c

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  $\Rightarrow$ marginal exiter more productive than avg. entrant

  $\Rightarrow$ marginal increase in the long-run M

Decomposition of the Elasticity - Y_3^{CES} to M_3

$$\underbrace{\underbrace{(q - q^{CES})}_{(a)} + \frac{\partial \log Y_3^{CES}}{\underbrace{\partial \log M_3}_{(b)}}}_{(2)}$$

- (a) is constant.

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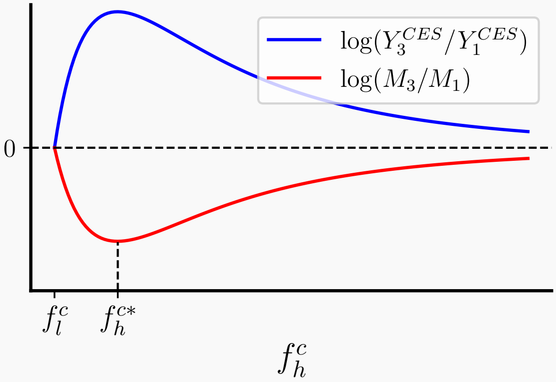
$$\underbrace{\underbrace{(q - q^{CES})}_{(a)} + \underbrace{\frac{\partial \log Y_3^{CES}}{\partial \log M_3}}_{(b)}}_{(2)}$$

- (a) is constant.
- (b) always negative but not monotone
  - when M_3 is large eliminating 1% of firms $\Rightarrow$ large labor savings
  - when M_3 is small $\Rightarrow$ small labor savings

Behaviour of CES output and varieties as a function of f_h^c



Interaction of Cycle Depth and Love-of-Variety

Taking stock:

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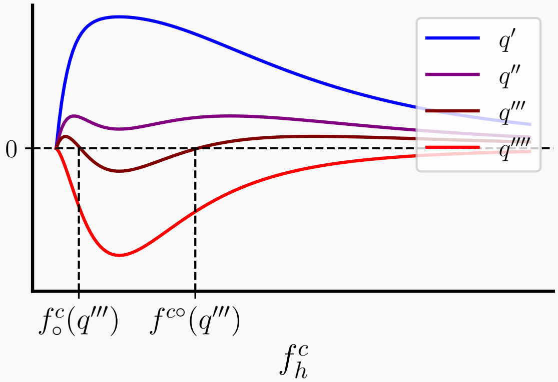

$$q' < q'' < q''' < q''''$$

Always good

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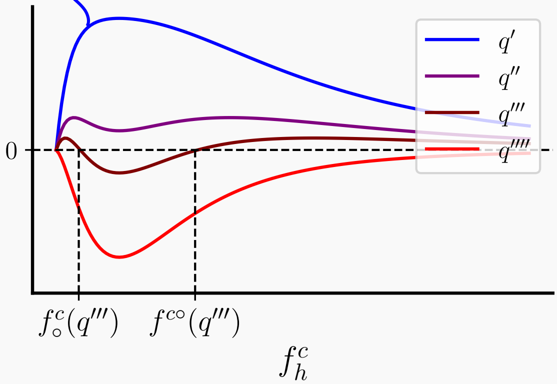

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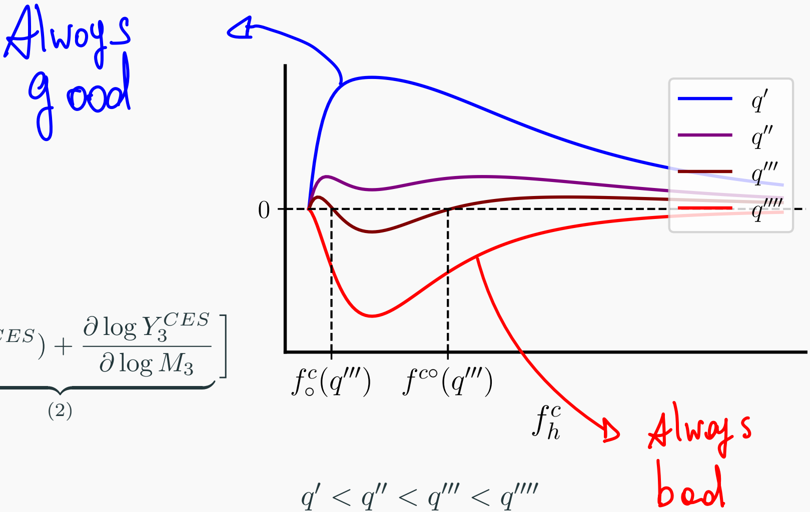

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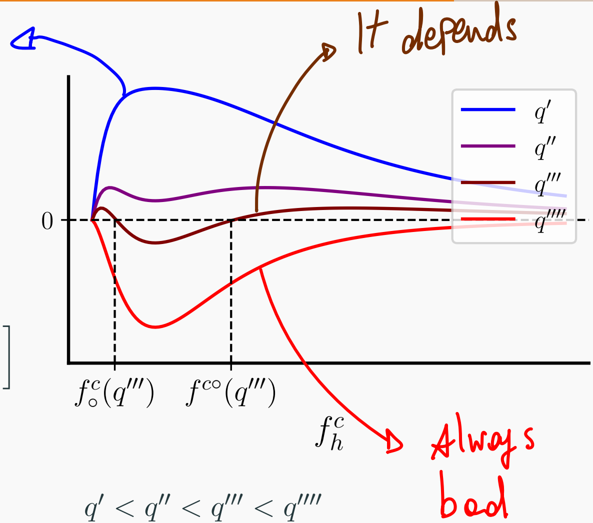

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Policy

Two Normative Results

Natural question: what should a planner do?

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Proposition

- *The economy is constrained efficient if and only if* $q = \frac{1}{\sigma-1}$.
- *Too few firms if* $q > \frac{1}{\sigma-1}$
- *Too many firms if* $q < \frac{1}{\sigma-1}$

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What should a planner do in recessions?

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Empirical Literature

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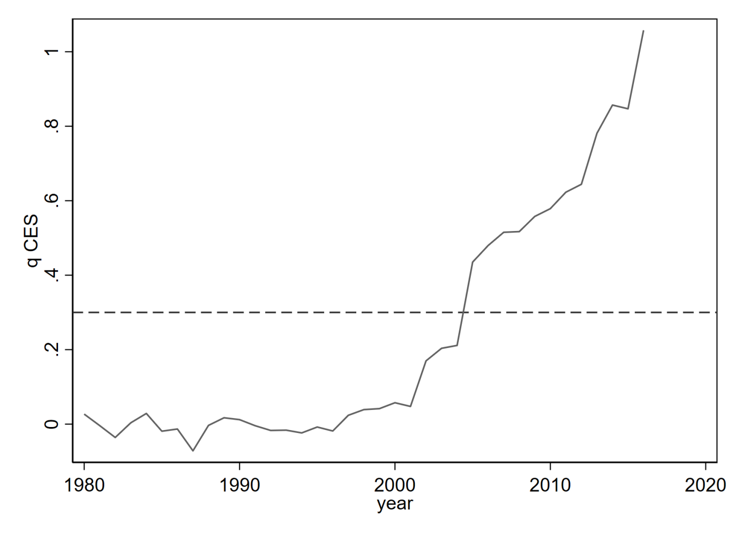
- We can recover σ for Belgium from aggregate markups over time

Relation to the Empirical Literature (2/2)



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- Multiproduct Firms
  - Product fixed cost cycles leave relative productivities unchanged ⇒ Entry and exit choices are unaffected ⇒ No Long-Run effects
  - Firm fixed cost shocks: same results with extra parameter: love-of-product-variety.

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- Even fixing a love-for-variety, some recessions may have long-run benefits while others do not.
- Same logic shapes the optimal policy conduct both in recessions and steady-state.

Thank you!

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Appendix

Danger of not considering external returns to scale

- Savagar and Kariel (2024) find evidence of both internal returns to scale and fixed costs increasing in the UK but aggregate productivity stagnating.
- They conclude that the stagnating aggregate productivity is likely to have been caused by increasing internal returns to scale + increase in markup.
- They rule out fixed cost story, but because they don't consider love-of-variety: fixed-cost increase + LoV would have also matched the stagnating TFP.

Dynamic Interpretation

Remark (Forward-Looking Firms)

Suppose that:

1. *Firms know that the time-t path of* $\{f_t^c\}_t$ *is weakly decreasing.*
2. *Firms calculate the present discounted value of their profit stream.*
3. *Firms receive one-shot offers on whether to enter. If they take the offer, they pay the fixed costs of entry* f^e*, draw their productivity* z*, and can then delay production until they become profitable.*

For large t*, the measure of firms in the economy is the same as in the case of myopic firms.*

Path Dependency

Remark (Path Dependence)

The stationary steady-state equilibrium is path-dependent.

- We considered economies that, in phases 1 and 3, feature identical parameters. Nonetheless, they are characterized by different equilibrium allocations.
- This property is fully driven by the presence of incumbents.

Path Dependency of Recessions

Remark (Path Dependence of Recessions)

Let a q-economy experience two cycles of the same intensity f_c^h. Then:
a) the post-crisis distribution, output, and welfare are different across the two cycles;
b) the recessions generate different degrees of cleansing, captured by $\underline{z}_2^1 \leq \underline{z}_2^2$, where $i \in \{1, 2\}$ denote the cycle.

- Having experienced the first recession, the q-economy starts the second crisis with a different incumbent population.
- During downturns, the composition of incumbents determines the cutoff for a given increase in the fixed cost f_c^h.